The case for the strong and conditional analysis of permission[1]

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Abstract. The classic analysis of permission modals involves existential quantification over a modal base. This analysis fails to account for many parallels between conditionals and permission, including non-monotonic and order-sensitive behavior in the scope of permission. An alternative-sensitive strict dynamic homogeneous conditional account of permission is given, which captures the parallels with conditionals, while also delivering simple and complex free choice patterns semantically. The semantic derivation of free choice makes it easier for the experimental evidence for the asymmetry between free choice and scalar implicatures to be accounted for than under the implicature approach to free choice.

Keywords: permission, free choice, conditional, strict analysis of conditionals, dynamic analysis of conditionals, strong permission

1. Introduction

Permission modals are usually analyzed as existential quantification over a modal base ($\mathcal{D}$ maps each w to its modal base, as below), possibly also with an ordering source, which I omit here:

(1) $\exists w' \in \mathcal{D}(w).\, p(w')$

However, according to the *deontic reduction* approach (Anderson, 1958; von Wright, 1971; Kanger, 1971), this is only one of two ways that permission is represented. There are both *weak* permission and *strong* permission:

(2) strong: $\Box(p \rightarrow \mathbf{OK}) \approx \forall w' \in \mathcal{E}(w).\, p(w') \rightarrow w' \in \mathcal{D}(w)$

(3) weak: $\Diamond(p \wedge \mathbf{OK}) \approx \exists w' \in \mathcal{E}(w).\, p(w') \wedge w' \in \mathcal{D}(w)$

OK is a proposition asserting the lack of deontic violations. $\Box$ and $\Diamond$ are epistemic modal operators (and so $\mathcal{E}(w)$ is w's epistemic modal base); deontic logic is then *reduced* to epistemic logic, hence the name *deontic reduction*. Strong permission as specified above is clearly too strong; it has roughly this meaning:

(4) In every epistemically accessible world, as long as p, then it is OK.

Notice that while too strong, strong permission in (2) has the property of validating free choice (FC, (7)), due to the validation of simplification of disjunctive antecedents (SDA, (8)) of material implication:

(5) $\Box((p \vee q) \rightarrow \mathbf{OK}) \Rightarrow \Box(p \rightarrow \mathbf{OK}) \wedge \Box(p \rightarrow \mathbf{OK})$

To capitalize on this property, Barker (2010) recasts strong permission under deontic reduction in linear logic, deriving FC. However, linear logic is a significant departure from mainstream semantic theory. Asher and Bonevac (2005), also aiming to derive FC, connect strong permission under deontic reduction to ordinary natural language conditionals:

(6) $p \;\Box\!\!\rightarrow \mathbf{OK}$.

[1]I would like to thank Kai von Fintel, Danny Fox, Viola Schmitt, and audiences at the Topics in Semantics seminar and SuB 29. All errors are mine.

Here and below, $p \boxright q$ is just a shorthand for the proposition denoted by *if p then q*, without committing to any specific analysis of conditionals.[2] Intuitively and in accordance with common analyses of conditionals (Kratzer, 2012; von Fintel, 2001), *if p then q* is weaker than strong permission in the deontic reduction tradition, because the worlds quantified over are restricted by a similarity relationship between worlds, rather than being all the epistemically accessible worlds. Therefore, (6) might be potentially more appropriate as a medium through which the meaning permission is understood. However, Asher and Bonevac (2005) postulates defeasible consequences of inference, and identifies FC inferences as such. Defeasible consequences also require non-trivial modifications to the architecture of semantics and pragmatics.

In this paper, I present data that highlights an array of analogies between permission and conditionals, beyond FC and SDA (section 2). The data further motivates an analysis of permission in terms of conditionals that can derive these analogies in a unified way. The proposal is simply that permission, i.e., *can p*, should be analyzed as $p \boxright$ **OK**; any empirically adequate analysis of the conditional, i.e., the $\boxright$ element, should suffice. Nevertheless, the preliminaries of a specific analysis in this fashion are sketched, based on ingredients already employed in the literature for the analysis of conditionals, among which the most important is the dynamic strict analysis of conditionals developed in von Fintel (2001). This analysis will require no significant departure from mainstream semantic architecture, unlike Barker (2010); Asher and Bonevac (2005). It will be shown that rather complex patterns of FC ($\Diamond\forall\vee$, Bar-Lev and Fox 2020; negative FC, Marty et al. 2021) follow straightforwardly from this preliminary strong and conditional analysis of permission. That FC is derived semantically also makes experimental evidence (Tieu et al., 2016, 2024; Chemla and Bott, 2014) that FC does not pattern with scalar implicatures easier to account for than with the dominant implicature approaches to FC (Fox, 2007; Bar-Lev and Fox, 2020).

2. Parallels between permission and conditionals

There are several parallels between the scope of permission modals and the antecedent of conditionals: SDA-FC, NPI, NSA, NWA, Sobel, R-Sobel, Multitude, CondPerm, and Homogeneity.

SDA-FC Disjunction that scopes inside becomes conjunction that scopes outside, known as free choice (FC) in permission (7) and simplification of disjunctive antecedents (SDA) in conditionals (8).

(7) You can read book one or book two.
 $\rightsquigarrow$ You can read book one.
 $\rightsquigarrow$ You can read book two.

(8) If you read book one or book two, you will pass the exam.
 $\rightsquigarrow$ If you read book one, you will pass the exam.
 $\rightsquigarrow$ If you read book two, you will pass the exam.

NPI NPIs are licensed in the scope of permission modals and the antecedent of conditionals:

(9) You can read any book.

[2] Asher and Bonevac (2005) insert one of the adverbs 'typically, generally, other things being equal, provided conditions are suitable' into the conditional schema for use in strong permission, but I think the restrictions on quantification that these elements impose are already implicit in ordinary natural language conditionals and do not need to be explicitly stated.

(10) If you read any book, you will pass the exam.

The resulting reading for both constructions is an SDA-FC-like wide-scope universal reading.[3]

NSA Both SDA-FC and NPI suggest a downward-entailing flavor to permission and conditionals. Yet, they are not downward-entailing enough to provide strengthening of the antecedent (SA).

(11) You can read book one.
  $\not\rightsquigarrow$ You can read book one and book two.
  $\not\rightsquigarrow$ You can read book one aloud.

(12) If you read book one, you will pass the exam.
  $\not\rightsquigarrow$ If you read book one but forget about all of its contents before the exam, you will pass the exam.

NWA Sometimes, there is also no *weakening* of conjunctive antecedents; this is much more common for conditionals. The relevant data for permission are reported in Williamson (2020); Kroedel (2023).

(13) If you read book one and book two, you will pass the exam.
  $\not\rightsquigarrow$ If you read book one, you will pass the exam.
  $\not\rightsquigarrow$ If you read book two, you will pass the exam.

(14) You can wear a swimsuit but also wear a suit over it at the formal dinner.
  $\not\rightsquigarrow$ You can wear a swimsuit at the formal dinner.

[3] An anonymous reviewer for Sinn und Bedeutung 29 comments that the NPI parallel is unwarranted, because *any* is 'ambiguous' between the NPI use and the FC use, and so the *any* in the scope of permission might not be a real NPI. Instead, they present *ever* as a real NPI, which is only licensed in the antecedent of conditionals but not in the scope of permission, casting doubt on the NPI parallel.

(i) *John can ever read a book.

There are several points that I would like to make in response. First, *ever* is also a special case because it is incompatible with the complements to permission, which are semantically predicates of events, yet *ever* arguably quantifies over times, and so an element containing *ever* will be too big semantically to fit in the scope of permission. Even when *ever* appears to be licensed after *can*, e.g., in the antecedent of conditionals and in the scope of negation, *ever* seems to outscope *can*:

(ii) If John can ever swim in this pool, he will be proud of himself.

Example (ii) means that 'If there is ever a time when John can swim in this pool, he will be proud of himself.'

Second, there are approaches to *any* that does not distinguish the two uses (Crnič, 2019, 2022). In general, if a downward-entailing condition is proposed for the licensing of NPIs, then it is also applicable to the licensing of *any* under *can*, because as soon as FC is derived, the scope of *can* is indeed downward-entailing with respect to the domain of *any*.

Third, there do seem to be NPIs that are not otherwise incompatible in the scope of permission but still fails to be licensed by permission, such as *single*:

(iii) *You can read a single book.

Compare (iii) with (iv):

(iv) If you read a single book, you will pass the exam.

Thus, (iii) might constitute genuine evidence that NPI might not be a true parallel between permission and conditionals. However, as examples (25) and (26) show, if conditional constructions are used canonically to express permission in natural language, an element of *even* might be observed. The relevant observation is that (iv) with *even* added no longer seems to license *single*:

(v) ??Even if you read a single book, you will pass the exam.

If permission involves *even*, either covertly or overtly, then it might be explained why *single* is not licensed in its scope. Nevertheless, I leave the comprehensive investigation of NPI licensing under permission to future research.

(15) In order to maintain peace, the US and the Soviet Union can both disarm.
$\not\rightsquigarrow$ The US can disarm.

Sobel, R-Sobel From NSA and NWA, Williamson (2020); Kroedel (2023) point to the similarity between NWA and Sobel sequences of conditionals. Here, I reproduce Sobel sequences directly with permission; they also show Reverse Sobel sequences (also known as Heim sequences), i.e., the felicity is dependent on the order of utterances and potentially point to the dynamicity of conditionals: they update the context non-intersectively.

Examples (16) and (17) illustrate Sobel and Reverse Sobel sequences with conditionals, with and without overt conjunction, respectively.

(16) Classic Sobel and Reverse Sobel sequences with conditionals
- a. If the US were to disarm, there would be war; if the US and the Soviet Union were to disarm, there would be peace.
- b. #If the US and the Soviet Union were to disarm, there would be peace; if the US were to disarm, there would be war.
- c. $p \Box\!\!\rightarrow \neg r; (p \wedge q) \Box\!\!\rightarrow r$
- d. $\#(p \wedge q) \Box\!\!\rightarrow r; p \Box\!\!\rightarrow \neg r$

(17) Sobel and Reverse Sobel without overt conjunction
- a. If I went to the store, it would be closed by the time I got there. But if I ran really fast to the store, it might of course still be open.
- b. ??If I ran really fast to the store, it might still be open. But if I went to the store, it would be closed by the time I got there.

von Fintel (2001)

Example (18) demonstrates that permission can interactive with conditionals in such sequences.

(18) Mixed conditional and permission Sobel and Reverse Sobel sequences
- a. If the US were to disarm, there would be war. But the US and the Soviet Union can both disarm.
- b. #If the US and the Soviet Union were to disarm, there would be peace. But the US cannot disarm.

Example (19) and (20) show that permission by itself enables the construction Sobel and Reverse Sobel sequences, with or without overt conjunction.

(19) Sobel and Reverse Sobel sequences with pure permission
In order to maintain peace,
- a. The US cannot disarm. But the US and the Soviet Union can both disarm.
- b. #The US and the Soviet Union can both disarm. But the US cannot disarm.
- c. $\neg\Diamond p; \Diamond(p \wedge q)$
- d. $\#\Diamond(p \wedge q); \neg\Diamond p$

(20) Sobel and Reverse Sobel with permission without overt conjunction
- a. You cannot wear a swimsuit at the formal dinner. But you can wear a swimsuit with a suit over it.
- b. ??You can wear a swimsuit with a suit over it at the formal dinner. But you cannot wear a swimsuit.

Sobel, in (19a) and (20a), is puzzling given the weak ∃-analysis of permission, because $\Diamond(p \wedge q)$ entails $\Diamond p$, as shown in (21), which will lead to a contradiction in (20a,b).

(21) $\exists w' \in \textsc{best}_o(\mathcal{D}(w)).\, p(w') \wedge q(w') \Rightarrow \exists w' \in \textsc{best}_o(\mathcal{D}(w)).\, p(w')$

R-Sobel is therefore not puzzling, but once an account derives the consistency of Sobel, a separate account is required to explain why R-Sobel is infelicitous. However, as we will see later, there are already accounts in the conditional literature that explains these patterns, which can and should be transferred to permission. Essentially, under the conditional view of permission, the Sobel and R-Sobel with permission is rewritten as

(22) a. $p \Box\!\!\rightarrow \neg\mathbf{OK}$; $(p \wedge q) \Box\!\!\rightarrow \mathbf{OK}$
b. $\#(p \wedge q) \Box\!\!\rightarrow \mathbf{OK}$; $p \Box\!\!\rightarrow \neg\mathbf{OK}$

This is then just an instance of Sobel and R-Sobel for conditionals; r in (16c) and (16d) is simply replaced by **OK**.

Multitude Lewis (1979) notes that permission seems to make permissible a multitude of worlds, as in (23):

(23) You are allowed to take Friday off.

Such worlds can correspond to the various ways the day off is spent. The intuition is not limited to examples like this, but inherent to any permission, especially the performative kind.[4] The classic ∃-analysis of permission cannot deliver this effect; the existence of one unspecified permissible world is asserted, but little more. On the other hand, conditionals, with their restricted universal force, can derive the desired effect. For example, if (23) is paraphrased into the following conditional,

(24) If you do no work on Friday, it will be fine.

the hearer might correctly understand that a multitude of worlds are permissible.[5]

CondPerm A rather straightforward correspondence between permission and conditionals is found when we widen the empirical scope to other languages. Permission is *canonically* granted in Korean (25) and Japanese (26) via conditional constructions (Chung, 2019; Kaufmann, 2017):

(25) John-un maykcwu-lul masi-eto toy-na-ta.
John-TOP beer-ACC drink-even.if GOOD-PRES-DECL
lit: 'Even if John drinks beer it's good.' ≈ 'John may drink beer.'

(26) tabe-te mo i-i.
eat-GER also good-NPST
lit.: 'If you eat it's also good.' ≈ 'You may eat.'

In English, while not canonically the case, permission can also be queried and granted via conditionals. (27) and (28) are very similar conversations.

[4] This is opposed to descriptive uses of permission, where effects of Multitude are possibly weaker; I will have nothing more to say for the distinction between the performative and the descriptive uses in this paper.

[5] See Klinedinst (2007) for a plural modification of the ∃-analysis that might approximate the effect. However, Schmitt (2023) presents arguments against the existence of world pluralities in semantics.

(27) A: Is it OK if I sleep?
B: Yes, it is OK if you sleep.

(28) A: May I sleep?
B: Yes, you may sleep.

It is thus reasonable or at least plausible to assume a conditional semantics for permission, if actual conditionals can be used for discourses about permission.

Homogeneity Once we accommodate the view that permission has a conditional format, then an additional parallel emerges. This parallel is homogeneity; negating a conditional is akin to negating just the consequent (von Fintel, 1997).

(29) Will this match light if I strike it? **No**.
a. ⇝ If I strike the match, it will not light.
b. $\neg(p \mathrel{\Box\!\!\rightarrow} q) \approx p \mathrel{\Box\!\!\rightarrow} \neg q$

(30) Only if you read book one$_{\text{F}}$ will you pass the test.
a. ⇝ You will not pass the test if you read book two.
b. The negative component of *only* produces homogeneity effects with the excludable alternatives.

This can be replicated with permission.

(31) Can I read book one? **No**.
a. ⇝ (If you read book one, then it would be not good.)
b. $\neg(p \mathrel{\Box\!\!\rightarrow} \textbf{OK}) \approx p \mathrel{\Box\!\!\rightarrow} \neg\textbf{OK}$

(32) You can only read book one$_{\text{F}}$.
a. ⇝ You cannot read book two. (If you read book two, then it will be not OK.)

However, this is also predicted given the classic ∃-analysis, because the ∃-analysis essentially treats the behavior under negation as the baseline semantics of permission. But if conditionals independently show homogeneity that cannot be accounted for by the baseline semantics, we might also just claim that the homogeneity effects of permission arise by being underlying conditionals.[6] The effect is made more dramatic when the permission or conditional involves disjunction, so that there is SDA-FC. The negation distributes to each disjunct, which then becomes a conjunct.

(33) Will I pass the test if I read book one or book two? **No**.
a. ⇝ If you read book one, you will not pass the test.
b. ⇝ If you read book two, you will not pass the test.
c. $\neg((p \lor q) \mathrel{\Box\!\!\rightarrow} r) \approx (p \mathrel{\Box\!\!\rightarrow} \neg r) \land (q \mathrel{\Box\!\!\rightarrow} \neg r)$

In permission, this is known as Double Prohibition (DP).

(34) Can I read book one or book two? **No**.
a. ⇝ You cannot read book one.
b. ⇝ You cannot read book two.
c. $\neg\Diamond(p \lor q) \approx \neg\Diamond p \land \neg\Diamond q$
d. $\neg(p \lor q) \mathrel{\Box\!\!\rightarrow} \textbf{OK} \approx (p \mathrel{\Box\!\!\rightarrow} \neg\textbf{OK}) \land (q \mathrel{\Box\!\!\rightarrow} \neg\textbf{OK})$

[6] However, see Bassi and Bar-Lev (2018) for an approach to conditionals where the base semantics is also existential and derives the universal flavor in UE environments via strengthening (exhaustification).

At this point, it is probably helpful to see how the classic ∃-analysis fares with the permission data at hand. The relevant results are summarized in Table 1. Because the ∃-analysis does not have a built-in mechanism to derive FC, I assume that one of the many accounts that specifically target FC is combined with it (Fox, 2007; Bar-Lev and Fox, 2020; Goldstein, 2019; Willer, 2018; Starr, 2016). In Table 1, ✓* indicates that the ∃-analysis combined with FC derives the phenomenon differently for conditionals and permission.

Table 1: Shared properties of conditionals and permission and predictions of the ∃-analysis w/ FC mechanism

	$\Box\!\!\rightarrow$	$\Diamond$	$\Diamond$ as ∃, w/ FC mechanism
SDA-FC	✓	✓	✓
NPI	✓	✓	✓
NSA	✓	✓	✓
NWA	✓	✓	✗
Sobel	✓	✓	✗
R-Sobel	✓	✓	✓
Multitude	✓	✓	✗
CondPerm	✓	✓	✗
Homogeneity	✓	✓	✓*

3. Implementation

As we have seen the many parallels between permission and conditionals, a unifying analysis is motivated. I will present a conditional analysis of permission in section 3.3 after setting up the appropriate background for the particular approach to conditionals employed, i.e., the dynamic strict analysis in von Fintel (2001), in sections 3.1 and 3.2.

3.1. Background: the conditional debate

In the classic, Kratzerian analysis, a conditional always involves a modal, which is universal unless there is an overt possibility modal, and there is an ordering over worlds based on similarity with the world of evaluation. Then, the modal quantifies over the closest (in terms of similarity) p-worlds to w (the world of evaluation), if the antecedent is p. Suppose that we are dealing with the universal flavor of conditionals, then we have the following:

(35) a. If p then q.
 b. $\forall w'.\ \textsc{best}_{\leq,w}(p).\ q(w')$

In (35b), $\leq$ maps each world w to the ordering based on similarity to w; w is the evaluation world; the actually ordering over worlds in this case is therefore $\leq_w$. This kind of quantification is naturally non-monotonic, and derives the consistency of Sobel, an example of which is reproduced in (37).

(36) $\forall w' \in \textsc{best}_{\leq,w}(p).\ \neg r(w')$ and $\forall w' \in \textsc{best}_{\leq,w}(p \wedge q).\ r(w')$ are consistent.

(37) If the US were to disarm, there would be war. But if the US and the Soviet Union were to both disarm, there would be peace.

The consistency of Sobel in conditionals is just like the consistency of the following pair involving superlatives; the analogy is due to Kai von Fintel.

(38) a. The closest gas stations are crummy.
b. But the closest Shell stations are great.

This style of analysis for the conditional is also known as *variably strict*, in contrast to the simply *strict* analysis, i.e., basically, material implication:

(39) a. If p then q.
b. $\forall w'. p(w') \rightarrow q(w')$

Sobel is a powerful argument against a simple strict analysis. Because with this semantics, there is downward monotonicity in the antecedent, and thus the inconsistency of Sobel cannot be avoided.

(40) $\forall w'. p(w') \rightarrow \neg r(w')$ entails $\forall w'. (p \wedge q)(w') \rightarrow \neg r(w')$.

However, R-Sobel becomes a major issue for variably strict semantics; if the Sobel sequence is consistent, why switching the order makes it the infelicitous Reverse Sobel sequence?

(41) #If the US and the Soviet Union both disarm, there would be peace. But if the US were to disarm, there would be war.

There is no common source of infelicity that can be detected in (41), like redundancy, presupposition failure, etc., according to the classic variably strict analysis of conditionals. R-Sobel prompted the revival and modification of the strict analysis in von Fintel (2001).

3.2. Dynamic strict analysis of conditionals

The idea is that Sobel is felicitous but R-Sobel is infelicitous because conditionals are *dynamic*; they change the context in non-intersective ways, and so order effects can arise. In particular, they change the *modal horizon* that persists in the conversation. The modal horizon can the thought of as an expanding sphere of epistemically accessible worlds. Given a conditional sentence $\varphi \mathrel{\Box\!\!\rightarrow} \psi$, the horizon h is updated in the following way in von Fintel (2001):

(42) $h[\varphi \mathrel{\Box\!\!\rightarrow} \psi]^{\leq} = \lambda w. h(w) \cup \left\{w' : \forall w'' \in [\![\varphi]\!]^{h,\leq} : w' \leq_w w''\right\}$

Essentially, all worlds as close to w as any φ-world is added to the horizon for w. An important property after the update is that the intersection of $h[\varphi \mathrel{\Box\!\!\rightarrow} \psi]^{\leq}(w)$ with $[\![\varphi]\!]^{h,\leq}$ is exactly the set of closest φ-worlds to w, *given that the horizon was not previously updated with a stronger antecedent than* φ. After the update, the sentence can be evaluated for truth:

(43) $[\![\varphi \mathrel{\Box\!\!\rightarrow} \psi]\!]^{h,\leq}(w) = 1$ iff
$\forall w' \in h[\varphi \mathrel{\Box\!\!\rightarrow} \psi]^{\leq}(w) \cap [\![\varphi]\!]^{h,\leq}. [\![\psi]\!]^{h[\varphi \Box\!\!\rightarrow \psi]^{\leq},\leq}(w')$

Essentially, there is simple universal quantification over the intersection of the updated horizon and the intension of φ evaluated in the horizon prior to the update; the assertion is that for each world in the intersection, the consequent evaluated with the updated horizon is true.

The starting horizon at the beginning of the discourse is assumed to be the singleton set containing just the evaluation world, $h_0(w) = \{w\}$. We can now verify that Sobel and R-Sobel are derived. With Sobel, the horizon after the update with *the US were to disarm* will contain all and only worlds as close to w as any *US-disarm*-world.

(44) If the US were to disarm, there would be war. But if the US and the Soviet Union were to both disarm, there would be peace.

Crucially, there are no *US-and-USSR-disarm*-worlds in the horizon, because it is assumed that these worlds are farther away than to w than the closest *US-disarm*-worlds. Thus, the first sentence can be judged true. The second sentence will then update the horizon with all and only worlds as close to w as any *US-and-USSR-disarm*-worlds. Thus, essentially, Sobel is derived just like in variably strict semantics. With R-Sobel, things are different.

(45) #If the US and the Soviet Union both disarm, there would be peace. But if the US were to disarm, there would be war.

The first sentence will directly expand the horizon to include worlds where both the US and the USSR disarm. When the second sentence is evaluated, the update is vacuous, and the intersection of the horizon with the antecedent will include *US-and-USSR-disarm*-worlds, which will make the second sentence false; the predicted judgment is that there should still be peace, rather than war, contrary to intuition. Thus, according to the dynamic strict approach to conditionals, Sobel is consistent, but R-Sobel is inconsistent; the infelicity is explained.

3.3. Defining permission as a conditional

This dynamic strict analysis of conditionals will then become the backbone of my account of permission. However, we are not only concerned with Sobel and R-Sobel; SDA-FC, NPI, Homogeneity should also follow. With these goals in mind, permission, or *can* for example, can be defined as follows:

(46) $[\![\text{can}]\!]^{w,h} = \lambda p : \mathsf{hom}(h'(w) \cap p, \lambda w'.\, \textsc{best}_{d(w)}(h'(w))(w')).$
$\forall w' \in h'(w) \cap p.\, \textsc{best}_{d(w)}(h'(w))(w'),$ where

a. $\mathsf{hom}(P_{\langle \alpha,t\rangle}, Q_{\langle \alpha,t\rangle}) := \forall a, a' \in P.\, Q(a) = Q(a')$

b. $h' = h[\textsc{alt}(p)]^{\leq} = h[\textsc{alt}(p)_1]^{\leq}[\textsc{alt}(p)_2]^{\leq} \ldots [\textsc{alt}(p)_n]^{\leq}$, i.e., successively updating h with the alternatives of p:
→ $h[\varphi]^{\leq} = \lambda w.\, h(w) \cup \{w' : \forall w'' \in [\![\varphi]\!]^{h,\leq} : w' \leq_w w''\}$[7]

c. $d(w)$ is w's deontic ordering

While (46) seems like a complex definition, it is simply the denotation of the conditional, given the dynamic strict analysis augmented with the homogeneity presupposition and an alternative-sensitive update of the modal horizon.

(47) a. If p, then it is **OK** (deontically optimal).

b. $p \;\Box\!\!\rightarrow \lambda w'.\, \textsc{best}_{d(w)}(h'(w))(w')$

As a dynamic strict system based on von Fintel (2001), the analysis is naturally capable of deriving Sobel and R-Sobel, as discussed in section 3.2. Additionally, NPI is also derived just by the present proposal being based on von Fintel (2001), because the antecedent is Strawson-downward-entailing; relevant discussions are in von Fintel (1999). The update mechanism for

[7] The $[\cdot]$ notation for updating is polymorphic in that it can either take a set of propositions or a single proposition as the input. When the input is a set of propositions S, then $h[S]$ means that h is successively updated by each member of S.

the modal horizon in (46b) differs from the one in von Fintel (2001) only in that the horizon h is not only updated with the antecedent p, but also alternatives of p. As we will see in section 3.3.1, this revised update mechanism will be responsible for the derivation of SDA-FC. The idea of updating with alternatives, however, is not my own innovation. Chung (2019), when giving an analysis of Korean weak deontic necessity, sketches a von Fintel (2001)-style dynamic account where the modal horizon is updated by the alternatives of the antecedent. Crucially, he is not concerned with SDA-FC effects, yet this innovation of his readily lends itself to SDA-FC. Therefore, the account sketched in Chung (2019) provides independent support for my adoption of this alternative-sensitive update mechanism.

Can presupposes that either the intersection of the updated horizon $h'(w)$ and the prejacent are all deontically the best in $h'(w)$ or none of them are the best in $h'(w)$; this presupposition captures Homogeneity. A similar presupposition is also the strategy that von Fintel (1997) adopts to capture the homogeneity of bare conditionals. If defined, *can* then asserts that the intersection of the updated horizon $h'(w)$ and the prejacent are all deontically the best in $h'(w)$. It will be shown in section 3.3.3 that the homogeneity presupposition also enables the definition of strong necessity modals as duals of permission modals, with additional desirable properties such as licensing negative free choice (Marty et al., 2021).

3.3.1. Deriving SDA-FC

Assuming that the disjuncts are alternatives to a disjunction, then we can derive SDA-FC. (48) illustrates simple FC of the form $\Diamond(p \vee q)$:[8]

(48) $[\![\text{can}]\!]^{w,h}(p \vee q)$
$\Leftrightarrow \forall w' \in h'(w) \cap (p \vee q).\ \textsc{best}_{d(w)}(h'(w))(w')$
$\Leftrightarrow \forall w' \in (h'(w) \cap p) \cup (h'(w) \cap q).\ \textsc{best}_{d(w)}(h'(w))(w')$
$\Leftrightarrow \forall w' \in h'(w) \cap p.\ \textsc{best}_{d(w)}(h'(w))(w') \wedge \forall w' \in h'(w) \cap q.\ \textsc{best}_{d(w)}(h'(w))(w')$
$\Rightarrow [\![\text{can}]\!]^{w,h'}(p) \wedge [\![\text{can}]\!]^{w,h'}(q)$
where $h' = h[\textsc{alt}(p \vee q)] \supseteq h[p][q]$.

Notice that $h'[p] = h'[q] = h'$. This means that after the utterance of *can* $p \vee q$, the sentence as well as the updated contexts entail *can* p and *can* q. This is captured by *dynamic entailment* that von Fintel (2001) formulates based on a notion from Stalnaker (1976).

(49) *Dynamic entailment*
$\varphi_1, \ldots, \varphi_n \models_{\text{dynamic}} \psi$ iff for all contexts c,
$[\![\varphi_1]\!]^c \cap \ldots \cap [\![\varphi_n]\!]^{c|\varphi_1|\ldots|\varphi_{n-1}|} \subseteq [\![\psi]\!]^{c|\varphi_1|\ldots|\varphi_n|}$

Let us make several definitions:

[8] Below, the symbol $\Leftrightarrow$ between two formulas of the metalanguage indicates that the truth of the formula on its left guarantees the truth of the formula on the right and vice versa; the symbol $\Rightarrow$ indicates that the truth of the formula on its left guarantees the truth of the formula on the right. Such a disclaimer is necessary because $[\![\text{can}]\!]$ is homogeneous, and $\varphi \Leftrightarrow \psi$ does not mean that φ and ψ are totally equivalent.

(i) $\varphi \Leftrightarrow \psi$ iff $\varphi = 1 \leftrightarrow \psi = 1$
(ii) $\varphi \Rightarrow \psi$ iff $\varphi = 1 \rightarrow \psi = 1$

(50) a. $c \coloneqq h$
b. $c|\text{can } p| \coloneqq c[\textsc{alt}(p)]^{\leq} = h[\textsc{alt}(p)_1]^{\leq}[\textsc{alt}(p)_2]^{\leq} \ldots [\textsc{alt}(p)_n]^{\leq}$

Then, on the left-hand side of $\models_{\text{dynamic}}$, we have (51):

(51) $[\![\text{can } p \text{ or } q]\!]^c$

On the right-hand side of $\models_{\text{dynamic}}$, we have (52):

(52) $[\![\text{can } p \text{ and can } q]\!]^{c|\text{can } p \text{ or } q|} = [\![\text{can } p]\!]^{c|\text{can } p \text{ or } q|} \wedge [\![\text{can } q]\!]^{c|\text{can } p \text{ or } q|}$

One can verify that dynamic entailment is satisfied with these denotations on each side of the definition and that these denotations are what is derived in (48).

Probably, $\neg p$ and $\neg q$ should also be alternatives of $p \vee q$ (or in general, *can p* should involve update with the alternative $\neg p$); it would have defeated the purpose the giving permission if p and q are not at least as good as $\neg p$ and $\neg q$. However, without $\neg p$ and $\neg q$ as alternatives, this comparison is not made explicit in the assertion. On the other hand, whether $p \wedge q$ should also be an alternative (or whether $p \wedge q$ should be pruned from the alternative set for modal horizon update) depends on whether one wants to compute the scalar implicature that $\neg\Diamond(p \wedge q)$. If this scalar implicature is desired, then $p \wedge q$ must not be used to update the modal horizon, since otherwise the assertion contradicts the scalar implicature: the closest p-worlds and closest q-worlds are as good as the closest $p \wedge q$-worlds.

The derivation of SDA-FC detailed here has the property of being semantic rather than pragmatic (e.g., as an implicature). There is experimental and developmental evidence that FC is dissimilar to scalar implicatures, in its early acquisition (Tieu et al., 2016), truth value judgments in non-homogeneous contexts (Tieu et al., 2024), and low processing costs (Chemla and Bott, 2014). The results in Tieu et al. (2024) also specifically call for a homogeneous non-implicature approach to FC, and my present proposal is such an approach. While the prominent implicature approach to FC (Fox, 2007; Bar-Lev and Fox, 2020; Del Pinal et al., 2024) can be made compatible with some of the experimental results as suggested by Tieu et al. (2016, 2024), the approach developed here can deliver a more straightforward explanation of the experimental observations, in line with other semantic accounts of FC.[9]

[9] There is another kind of argument against an implicature approach to FC in Wehbe and Doron (2024): unlike scalar implicatures, FC inference does not show *Post Accommodation Informativity* effects.

(i) **Post-Accommodation Informativity** (PAI): A sentence S_p (presupposing p) can be uttered felicitously only if S_p is informative w.r.t. the common ground after accommodating p.

PAI follows from a Stalnakerian view of presupposition accommodation, and is supported by (ii):

(ii) I knew that all of John's kids are adopted, but today I discovered something interesting. . .
a. #All 5 of John's children are adopted!
b. John has 5 adopted kids!

Scalar implicatures are shown to pattern with presuppositions:

(iii) I knew that at least one of the students read the book, but today I learned something interesting. . .
a. #Some of the students read the book.
b. Not all of the students read the book.

On the other hand, FC does not show this effect:

(iv) I knew that Mary is at least allowed to eat cake, but today I learned something interesting. She's allowed to eat cake or ice-cream.

While my account does not derive FC as an implicature, there is nevertheless a homogeneity presupposition, whose accommodation in (iv) will lead to triviality of the assertion, predicting infelicity that is not borne out. My

3.3.2. Further FC cases

In this section, I show that the FC derivation delivers on other related patterns. In (53), I check that $\neg\Diamond(p \vee q)$ is well-behaved, i.e, Double Prohibition should be derived with the homogeneity presupposition:

(53)
$$\begin{aligned}
&\neg[\![\text{can}]\!]^{w,h}(p \vee q)\\
\Leftrightarrow\ & \forall w' \in h'(w) \cap (p \vee q).\ \neg\textsc{best}_{d(w)}(h'(w))(w')\\
\Leftrightarrow\ & \forall w' \in (h'(w) \cap p) \cup (h'(w) \cap q).\ \neg\textsc{best}_{d(w)}(h'(w))(w')\\
\Leftrightarrow\ & \forall w' \in h'(w) \cap p.\ \neg\textsc{best}_{d(w)}(h'(w))(w') \wedge \forall w' \in h'(w) \cap q.\ \neg\textsc{best}_{d(w)}(h'(w))(w')\\
\Rightarrow\ & \neg[\![\text{can}]\!]^{w,h'}(p) \wedge \neg[\![\text{can}]\!]^{w,h'}(q)
\end{aligned}$$
where $h' = h[\textsc{alt}(p \vee q)] \supseteq h[p][q]$.

Additionally, the current approach to SDA-FC can directly handle a case where the implicature approach must adopt the *Innocent Inclusion* modification in Bar-Lev and Fox (2020): $\Diamond\forall\vee$-FC, which is schematized in (54) and exemplified in (55):

(54) $\Diamond\forall x.\, P(x) \vee Q(x) \rightsquigarrow (\Diamond\forall x.\, P(x)) \wedge (\Diamond\forall x.\, Q(x))$

(55) John is OK with every student singing or dancing.
  a. $\rightsquigarrow$ John is OK with every student singing.
  b. $\rightsquigarrow$ John is OK with every student dancing.

In the present approach, as long as we update the modal horizon with the expected alternatives, the FC inference is generated:

(56)
$$\begin{aligned}
&[\![\text{can}]\!]^{w,h}(\forall x.\, P(x) \vee Q(x))\\
\Leftrightarrow\ & \forall w' \in h'(w) \cap (\forall x.\, P(x) \vee Q(x)).\ \textsc{best}_{d(w)}(h'(w))(w')\\
\Rightarrow\ & \forall w' \in h'(w) \cap (\forall x.\, P(x) \vee \forall x.\, Q(x)).\ \textsc{best}_{d(w)}(h'(w))(w')\\
\Leftrightarrow\ & \forall w' \in (h'(w) \cap \forall x.\, P(x)) \vee (h'(w) \cap \forall x.\, Q(x)).\ \textsc{best}_{d(w)}(h'(w))(w')\\
\Leftrightarrow\ & (\forall w' \in (h'(w) \cap \forall x.\, P(x)).\ \textsc{best}_{d(w)}(h'(w))(w')) \wedge\\
& (\forall w' \in (h'(w) \cap \forall x.\, Q(x)).\ \textsc{best}_{d(w)}(h'(w))(w'))\\
\Rightarrow\ & [\![\text{can}]\!]^{w,h'}(\forall x.\, P(x)) \wedge [\![\text{can}]\!]^{w,h'}(\forall x.\, Q(x))
\end{aligned}$$
where $h' = h[\textsc{alt}(\forall x.\, P(x) \vee Q(x))] \supseteq h[\forall x.\, P(x)][\forall x.\, Q(x)]$.

Notice that $h'[\forall x.\, P(x)] = h'[\forall x.\, Q(x)] = h'$.

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conjecture is that FC is akin to overt conjunction, which while displaying homogeneity (*I saw John and Mary* $\rightsquigarrow$ *I saw John and I saw Mary*; *I didn't see John and Mary* $\rightsquigarrow$ *I didn't John and I didn't see Mary*), does not give rise to PAI effects, unlike the semantically very similar definite plurals, which are also homogeneous:

(v) I knew that at least one of the professors laughed. Today I learned something interesting...
  a. #The professors laughed.
  b. All the professors laughed.

(vi) I knew that at least John came to the party. Today, I learned something interesting...
  a. John and Mary came to the party.
  b. Both John and Mary came to the party.

Free choice disjunction, by making the disjuncts explicitly available, obviates PAI effects, just like in overt conjunction, even though both are homogeneous and known information makes the post-accommodation assertion trivial.

3.3.3. Duality, *need*, and negative FC

In the present approach, strong necessity deontic modals like *need* or *must* (using *need* as a representative below) can be defined as the dual of *can*, as in (57); by the homogeneity presupposition, we get exactly the familiar semantics of *need* in the affirmative case, although augmented with alternative-sensitivity and dynamicity.

(57) $[\![\text{need}]\!]^{w,h}(p) \coloneqq \neg[\![\text{can}]\!]^{w,h}(\neg p)$

(58)
$$\begin{aligned}[\![\text{need}]\!]^{w,h}(p) &\Leftrightarrow \neg[\![\text{can}]\!]^{w,h}(\neg p)\\ &\Leftrightarrow \forall w' \in h'(w) \cap \neg p.\ \neg\textsc{best}_{d(w)}(h'(w))(w')\\ &\Leftrightarrow \forall w' \in \textsc{best}_{d(w)}(h'(w))(w').\ w' \notin (h'(w) \cap \neg p)\\ &\Leftrightarrow \forall w' \in \textsc{best}_{d(w)}(h'(w))(w').\ w' \notin h' \vee w' \notin \neg p\\ &\Leftrightarrow \forall w' \in \textsc{best}_{d(w)}(h'(w))(w').\ w' \in p\end{aligned}$$
where $h' = h[\textsc{alt}(\neg p)] \supseteq h[p][\neg p]$.

On the other hand, the negation of *need* acquires new properties. The meaning is illustrated in (59):

(59)
$$\begin{aligned}\neg[\![\text{need}]\!]^{w,h}(p) &\Leftrightarrow \neg\neg[\![\text{can}]\!]^{w,h}(\neg p)\\ &\Leftrightarrow [\![\text{can}]\!]^{w,h}(\neg p)\\ &\Leftrightarrow \forall w' \in h'(w) \cap \neg p.\ \textsc{best}_{d(w)}(h'(w))(w')\end{aligned}$$
where $h' = h[\textsc{alt}(\neg p)] \supseteq h[p][\neg p]$.

Essentially, $\neg[\![\text{need}]\!](p)$ says that 'you are allowed to not do p.' Because it is the same as *can not*, we easily derive *negative free choice* (Marty et al., 2021): inferences of the form $\neg\Box(p \wedge q) \rightsquigarrow \neg\Box p \wedge \neg\Box q$.

(60) Mary is not required to read both book one and book one.
  a. $\rightsquigarrow$ Mary is not required to read book one.
  b. $\rightsquigarrow$ Mary is not required to read book one.

Example (61) shows that negative FC is a dynamic entailment.

(61)
$$\begin{aligned}&\neg[\![\text{need}]\!]^{w,h}(p \wedge q)\\ \Leftrightarrow\ &\forall w' \in h'(w) \cap \neg(p \wedge q).\ \textsc{best}_{d(w)}(h'(w))(w')\\ \Leftrightarrow\ &\forall w' \in h'(w) \cap (\neg p \vee \neg q).\ \textsc{best}_{d(w)}(h'(w))(w')\\ \Leftrightarrow\ &\forall w' \in (h'(w) \cap \neg p) \cup (h'(w) \cap \neg q).\ \textsc{best}_{d(w)}(h'(w))(w')\\ \Leftrightarrow\ &\forall w' \in h'(w) \cap \neg p.\ \textsc{best}_{d(w)}(h'(w))(w') \wedge \forall w' \in h'(w) \cap \neg q.\ \textsc{best}_{d(w)}(h'(w))(w')\\ \Rightarrow\ &\neg[\![\text{need}]\!]^{w,h'}(p) \wedge \neg[\![\text{need}]\!]^{w,h'}(q)\end{aligned}$$
where $h' = h[\textsc{alt}(\neg(p \wedge q))] \supseteq h[\neg p][\neg q]$.

Notice that $h'[\neg p] = h'[\neg q] = h'$. (62) is the sanity check for $\neg\Box(p \vee q)$:

(62)
$$\begin{aligned}&\neg[\![\text{need}]\!]^{w,h}(p \vee q)\\ \Leftrightarrow\ &\forall w' \in h'(w) \cap \neg(p \vee q).\ \textsc{best}_{d(w)}(h'(w))(w')\\ \Leftrightarrow\ &\forall w' \in h'(w) \cap (\neg p \wedge \neg q).\ \textsc{best}_{d(w)}(h'(w))(w')\end{aligned}$$
where $h' = h[\textsc{alt}(\neg(p \vee q))] \supseteq h[\neg p \wedge \neg q]$.

This claims that worlds as close as the closest $\neg p \wedge \neg q$-worlds are the best; in $\neg[\![\text{need}]\!](p \wedge q)$, these worlds are not even quantified over, i.e., not updated into the modal horizon in the first place.

4. Discussion

The proposal here is still preliminary. I do not have an explicit definition of the set of alternatives that can update the modal horizon, but only minimal requirements for the derivations here. There is evidence that negative FC and the analogous simplification of negated conjunctive antecedents (SNCA) might be qualitatively different from FC and SDA (Marty et al., 2021; Ciardelli et al., 2018), but they are derivable in basically the same way in this approach. There is also debate on the correctness of the strict dynamic approach, e.g., Boylan and Schultheis (2021: against, a.o.) and Greenberg (2021: for, a.o.). The details of the implementation of the proposal are thus subject to change due to future resolution of such debates.

Nevertheless, the paper still advances the argument that a strong, conditional analysis of permission will better capture the analogies between permission and conditionals (SDA-FC, NPI, Sobel, R-Sobel, and Homogeneity) and the remarkable properties of permission (Multitude and CondPerm). Such an analysis has the potential to derive the empirical facts about permission with existing tools for the analysis of conditionals, which is desirable from the perspective of theoretical economy. The straightforward extension of existing components in conditional analysis to permission sketched in this paper also already delivers on the more intricate cases of FC, such as negative FC and $\Diamond\forall\vee$-patterns. Finally, it provides a semantic rather than pragmatic (implicature-based) derivation of FC, and therefore a simpler explanation of experimental evidence for the asymmetry between FC and scalar implicatures than is possible with the implicature approach to FC.

Compared to the previous approaches to FC that relies on the concept of strong permission, i.e.,Asher and Bonevac (2005); Barker (2010), the present approach is more conservative in the implementation. There is no need for a different logic (linear logic, Barker 2010), nor the need for entailments to be defeasible (Asher and Bonevac, 2005). All of the formal ingredients in the present approach also stem from well-established approaches, such as the dynamic strict analysis of conditionals, alternative-sensitivity, and homogeneity presuppositions.

Whether the empirical observations and analytic approach can be carried over to other kinds of modals, e.g., epistemic possibility modals and weak necessity deontic modals, will require further investigation.

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